# Climate of Minnesota

## Part VI

## SOLAR RADIATION AT ST. PAUL

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Technical Bulletin 280 - 1971
Agricultural Experiment Station
University of Minnesota

## Contents



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## Introduction

Agriculture, which may be defined as the science and art of capturing solar energy within plant and animal tissues, is the only major industry that uses solar energy continually and directly. New and interesting uses within agriculture for which solar radiation information currently is required include evapotranspiration calculations and mathematical models that predict light interception and photosynthetic rates.

It is very possible that other industries, particularly the power producers, soon may find themselves forced by public or economic pressure to look to energy sources other than fossil fuels or atomic energy. If solar energy is not used as a direct source in some areas due to uncertainties caused by cloudy conditions, then it might become a secondary source through temporary storage in cells.

Therefore, with the idea that solar energy will receive continued attention from agriculture and renewed attention as a power source, this bulletin presents detailed data on the theoretical and actual amounts of solar radiation received on a horizontal surface at St. Paul, Minnesota. Others for whom this information is intended include architects, builders, heating and air conditioning engineers, and any others who require solar radiation data.

## Source of Data

The data shown include the daily march of (a) the extraterrestrial radiation, (b) the average clear-day direct plus diffuse radiation measured at the earth's surface, and (c) the average-day total radiation measured at the earth's surface. Both (b) and (c) are shown for the middle day of each climatological week and the extraterrestrial radiation shown is that calculated for the 21st day of each month. In the climatological year, week number one is March 1-7, and the 52nd week is February 21-27. Week 53, for which no information is shown, consists of February 28 and 29. The advantage of this scheme is that every week of the year has the same 7 dates whether it is a leap year or not.

Extraterrestrial radiation is the amount of radiation received at the outer limit of the earth's atmosphere before scattering and absorbtion of the radiation occur. The amount of radiation just outside the atmosphere's influence incident on a surface perpendicular to the sun's rays, termed the "solar constant," is not yet firmly established. In this bulletin all extraterrestrial radiation data were calculated assuming a "solar constant" of 1.94 cal $cm^{-2}$ $min^{-1}$ with appropriate corrections for latitude and distance from the sun.

The remaining data, clear-day direct plus diffuse, clear-day direct, clear-day diffuse, and average-day direct plus diffuse solar radiation, are all based upon values measured at the earth's surface.

Solar radiation reaching the earth's surface is made up of two distinct parts, direct beam radiation and diffuse radiation. Upon entering the earth's atmosphere, solar radiation encounters air molecules which scatter the light. As long as the scattering particles are much smaller than the wavelength of light, the shorter wavelengths (blue) are scattered more than the longer wavelengths (red). The direct beam sunlight is further scattered by dust and vapor in the atmosphere. This scattered light, which is richer in shorter wavelengths than is direct sunlight, is termed diffuse light or skylight. It is this diffuse radiation that provides light on the side of obstructions shaded from direct sunlight. Clouds, too, alter the direct beam by reflecting a portion and transmitting another. As a result, diffuse light can vary from 100 percent on an overcast day to a small fraction of the total on a clear day.

The measured values of solar radiation in this study were obtained from the 1965-1969 records of the Agricultural Weather Station maintained by the Soil Science Department of the Institute of Agriculture. This station is located at 44° 59′ N and 93° 11′ W at the elevation of 969 ft. msl.

The solar radiometers used were 50-junction pyranometers with their output recorded on a circular-chart single pen recorder. The calibration of the pyranometers was checked against a similar instrument held in reserve for this purpose. The recorder was serviced by the manufacturer's service department at least twice per year, and it also was checked with a standard potentiometer at more frequent intervals. An observer was at the weather station at least once per day, and the pyranometer bulb was cleaned as required.

## Construction of Figures

Hourly values of the extraterrestrial radiation were calculated for the 21st day of each month (1). For this reason, the extraterrestrial radiation is shown for approximately every 4th week rather than weekly.

The clear-day values were reconstructed from records when the day or part of the day was free of visible clouds. The hourly values obtained were charted, smoothed, and plotted.

Values for the diffuse radiation were obtained by shielding the pyranometer from the direct rays of the sun with a small blackened paddle held about 6 feet from the radiation sensor during cloud-free periods. Over a period of 5 years, this was done frequently enough at various times during the day that the intensity of the diffuse radiation could be constructed for an average clear-day within each climatological week.

The average-day radiation for each climatological week was obtained by calculating the mean radiation received at 0600 local standard time (6 A.M.) and at 2-hour intervals thereafter for each day of the 5-year period. These values were plotted and a smooth line drawn to connect the 2-hourly mean values.

In those weeks where the extraterrestrial values are plotted, it is apparent that the measured solar radiation day is shorter than the extraterrestrial day. This is because the pyranometers used are insensitive to the solar rays when the sun is within a few degrees of the horizon. As a result, the measured values indicate an apparent sunrise about 15-30 minutes later and a sunset about 15-30 minutes earlier than is actually the case.

## Solar Time and Daylength

The extraterrestrial hourly values were plotted according to solar time rather than the local standard time, which is shown on the horizontal axis in figure 6. The measured radiation values were plotted for the appropriate local standard time.

Because local standard time varies about solar noon during the course of a year, the maximum daily radiation values seldom coincide with local noon. There are two reasons for this variation between solar time and local standard time. This may be demonstrated by considering solar noon, defined as the time when the sun is directly in the south. St. Paul is not in the center of the central time zone but in the western portion. This causes a constant factor of about 12 minutes delay before the earth has turned enough for the longitude of St. Paul (93° 11′ W) to be directly under the sun. In addition, this constant longitude factor is modified by a variable time factor called "the equation of time." The latter is caused by the elliptical orbit of the earth about the sun, and the angle of the earth's axis to the ecliptic plane. In figure 1 the two factors have been combined to show when the sun is directly in the south (solar noon) according to central standard time. The greatest difference between solar time and local standard time occurs in the week of February 10 when solar noon is 27 minutes after local noon. Only in mid-October and late November do local standard noon and solar noon coincide.

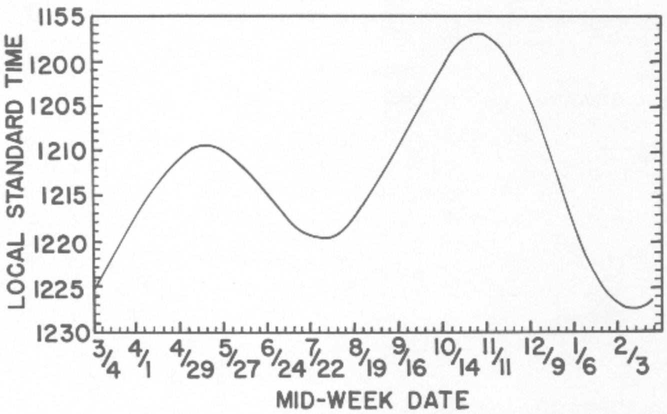



Figure 1. The occurrence of solar noon at St. Paul (93° 11′ W) in terms of central (local) standard time.

Daylength is very important with respect to solar radiation total values. Figure 2 shows the sunrise and sunset time at St. Paul for an average year. The variation in length of day is quite great, the longest day occurring at the time of the summer solstice (June 22) and the shortest at the winter solstice (December 22). There is a 6 hour, 51 minute difference between these 2 days. The variation in daylength is greater farther north. For example, Grand Forks, nearly 40°N, on June 22 is 26 minutes longer and on December 22 is 24 minutes shorter than indicated in figure 2.

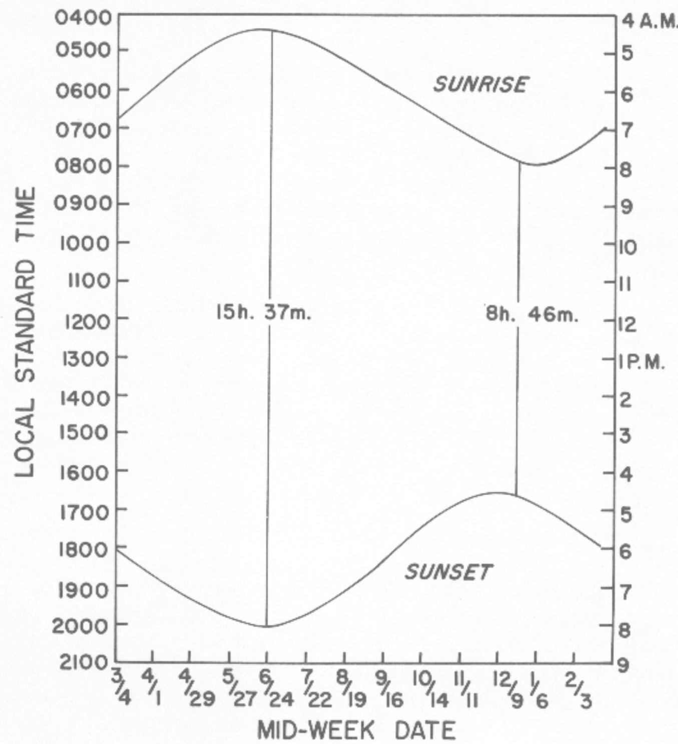



Figure 2. Time of sunrise and sunset at St. Paul-Minneapolis. The duration in hours and minutes of the longest and shortest days of the year are shown also.

## The Apparent Solar Position

The word apparent is used in the title of this section because to an earth-bound observer it appears that the sun moves around him. In reality, of course, the earth, while rotating once per 24 hours on its axis, revolves about the sun in an orbit that takes about 365¼ days to complete. The daily rotation produces the day-night effect and the revolution about the sun results in the seasonal effect.

Figures 3 and 4 are drawn as if the observer were on a stable platform (the earth) with the sun moving about him. Figure 3 shows the altitude of the sun above the horizon at solar noon at St. Paul. For stations north of 45° the solar altitude at any time of the year is simply 1° lower than shown in figure 3 for each degree beyond St. Paul. Similarly stations south of St. Paul have a noon solar altitude 1° higher than shown for each degree of latitude south of 45°N.

Figure 3 may find application in architectural and landscape design and perhaps elsewhere. One question that can be answered is the angle to which a roof must be pitched so that no direct solar rays are received on it, another might be the amount of shade cast by a tree or building at any time of the year. For example, on March 21 and September 23 when the noon solar altitude is 45° at St. Paul, a roof pitched 45° or more to the north will receive no direct rays of the sun. In contrast, a roof pitched 45° to the south will be perpendicular to the noon rays of the sun.

The shadow cast by an object of known height by the sun can be determined easily by trigonometry. As

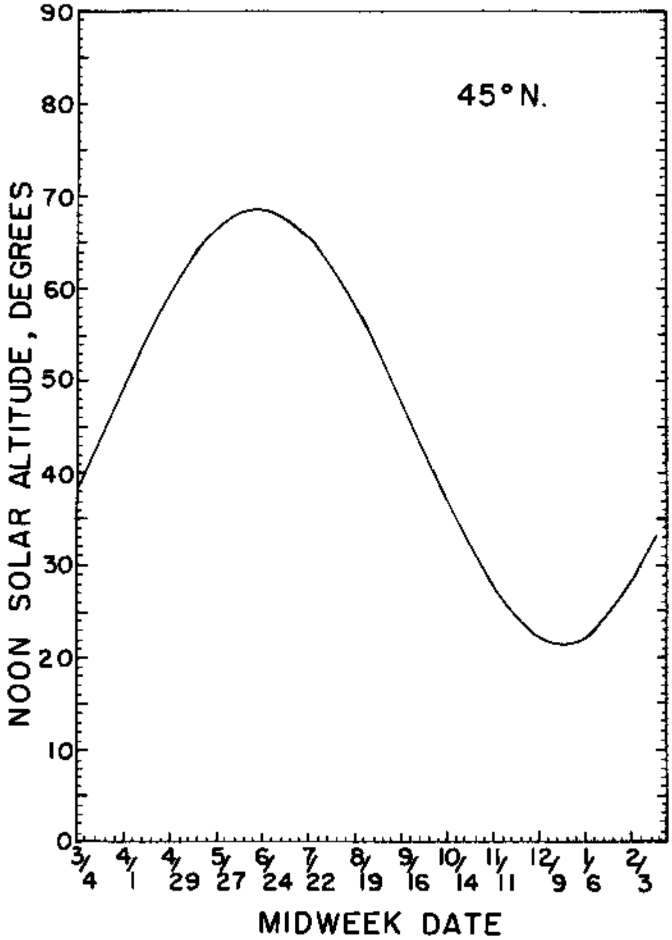



Figure 3. Altitude of the sun above the horizon at solar noon at St. Paul.

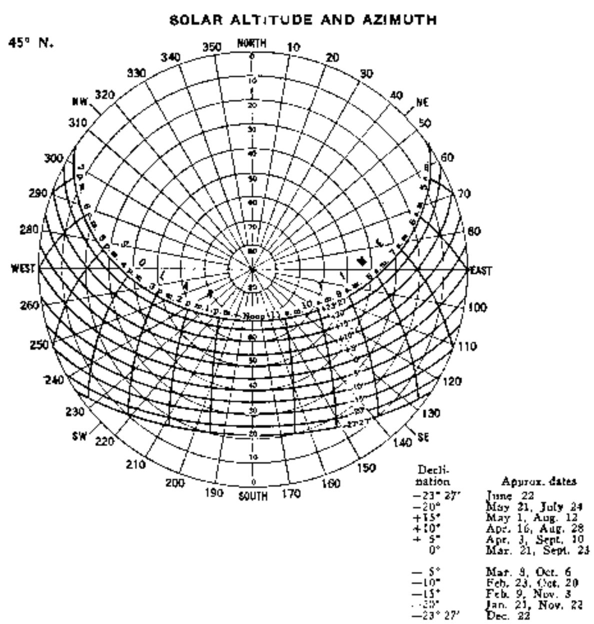



Figure 4. The apparent path of the sun in the sky during selected dates. See literature cited (4).

an example, a building 100 feet tall will cast a shadow at noon on June 22 that will be 39 feet long (shadow length = 100' ÷ tangent of the noon solar altitude angle, i.e., tan 68.5°), but 254 feet long on December 22 (shadow length = 100' ÷ tangent of the noon solar altitude angle, i.e., tan 21.5°).

Methods for calculating the amount of radiation received on a surface of any orientation and slope are found in Brooks (2) and Heywood (3).

Figures 3 and 4 provide the data necessary for all of these calculations.

Figure 4 shows the apparent path that the sun traces in the sky during the year at 45°N. This figure shows the solar altitude and azimuth (direction from north) from sunrise to sunset at any hour of the day for the indicated dates. The solar path at other times of the year may be obtained from the figure by interpolation. The chart shows, for example, that on June 22 (solar declination of +23° 27') the sunrise azimuth is 56° and the sunset azimuth is 304°. In contrast, on December 22 (solar declination of —23° 27') the sun rises at 124° and sets at 236° (approximately SE and SW, respectively). It also is shown that at 10 solar time on June 22 the altitude and azimuth of the sun will be 57° and 122°, respectively.

The eaves of a building are important with respect to the sun's rays. On a south-facing building, the eaves may be extended so that only the less intense early morning and late afternoon rays enter a south-facing window, figure 5, while the winter solar rays can enter the window without restriction. Such a design has the obvious advantage of decreasing both the heat load in summer and the heat requirement in winter.

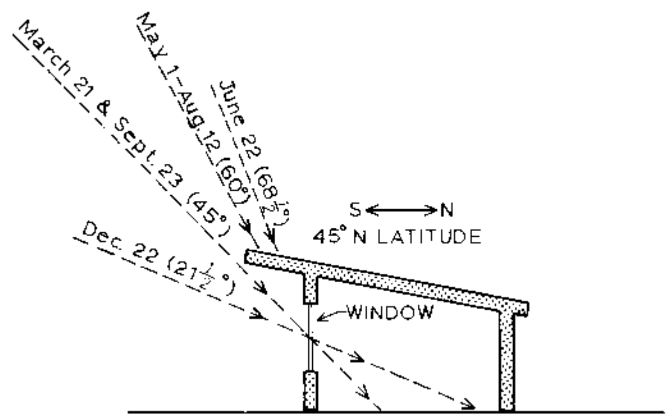



Figure 5. Building constructed so that direct rays of the noontime sun between May 1 and August 12 do not enter the window. Altitude of the sun above the horizon at noon is shown for indicated dates at 45°N latitude (St. Paul); the solar altitude ranges from a maximum of 68 1/2° on June 22 to a minimum of 21 1/2° on December 22.

## Discussion

Figure 6 shows the average daily march of solar radiation from sunrise to sunset for the middle day of each climatological week of the year (except the extraterrestrial radiation which was calculated for the 21st day of each month). For example, in the week of March 15-21 the solar noon value of an average clear-day at the earth's surface will equal about 1.11 cal $cm^{-2}$ $min^{-1}$ and the diffuse about 0.20 cal $cm^{-2}$ $min^{-1}$. Thus, the diffuse radiation constitutes about 18 percent of the total received at the earth's surface at noon. The extraterrestrial noon-time value is 1.38 cal $cm^{-2}$ $min^{-1}$ on March 21. Because 1.11 cal $cm^{-2}$ $min^{-1}$ penetrates the atmosphere at noon on a clear day, the atmospheric transmission averages about 80 percent. At noon on a day of average cloudiness for this same week, 0.70 cal $cm^{-2}$ $min^{-1}$ is received, which is 63 percent of the clear-day noon value. The average-day noon atmospheric transmission in this week, therefore, is just a little more than 50 percent.

It should be apparent that for any given hour or day the radiation intensity may vary greatly from the hourly average-day values shown in figure 6. For example, comparison of the noon clear-day value in the week of March 15-21 shows 1.11 cal $cm^{-2}$ $min^{-1}$ against the average-day noon value of 0.70 cal $cm^{-2}$ $min^{-1}$. Thus, the average-day noon value can vary by more than 58 percent simply due to the absence of clouds.

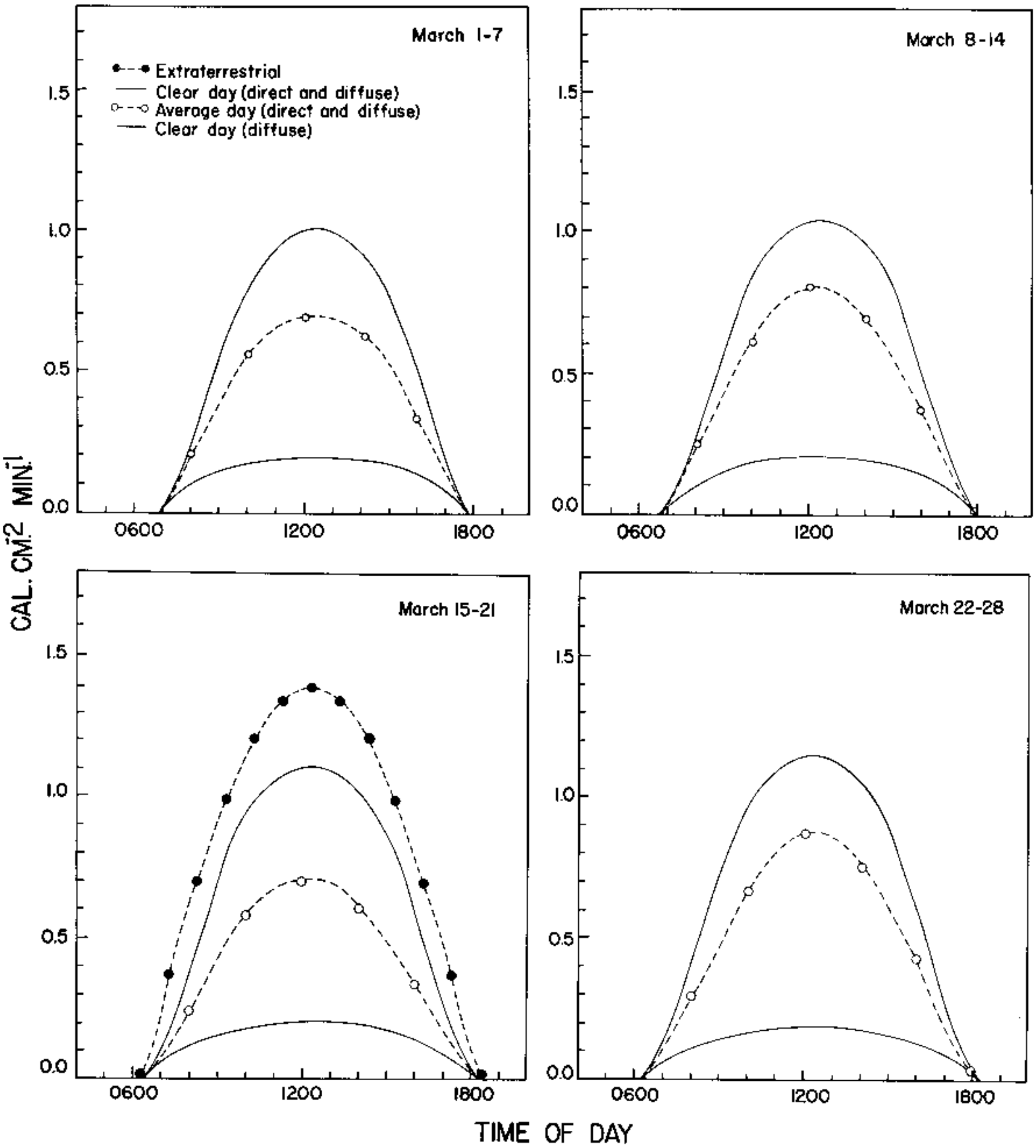
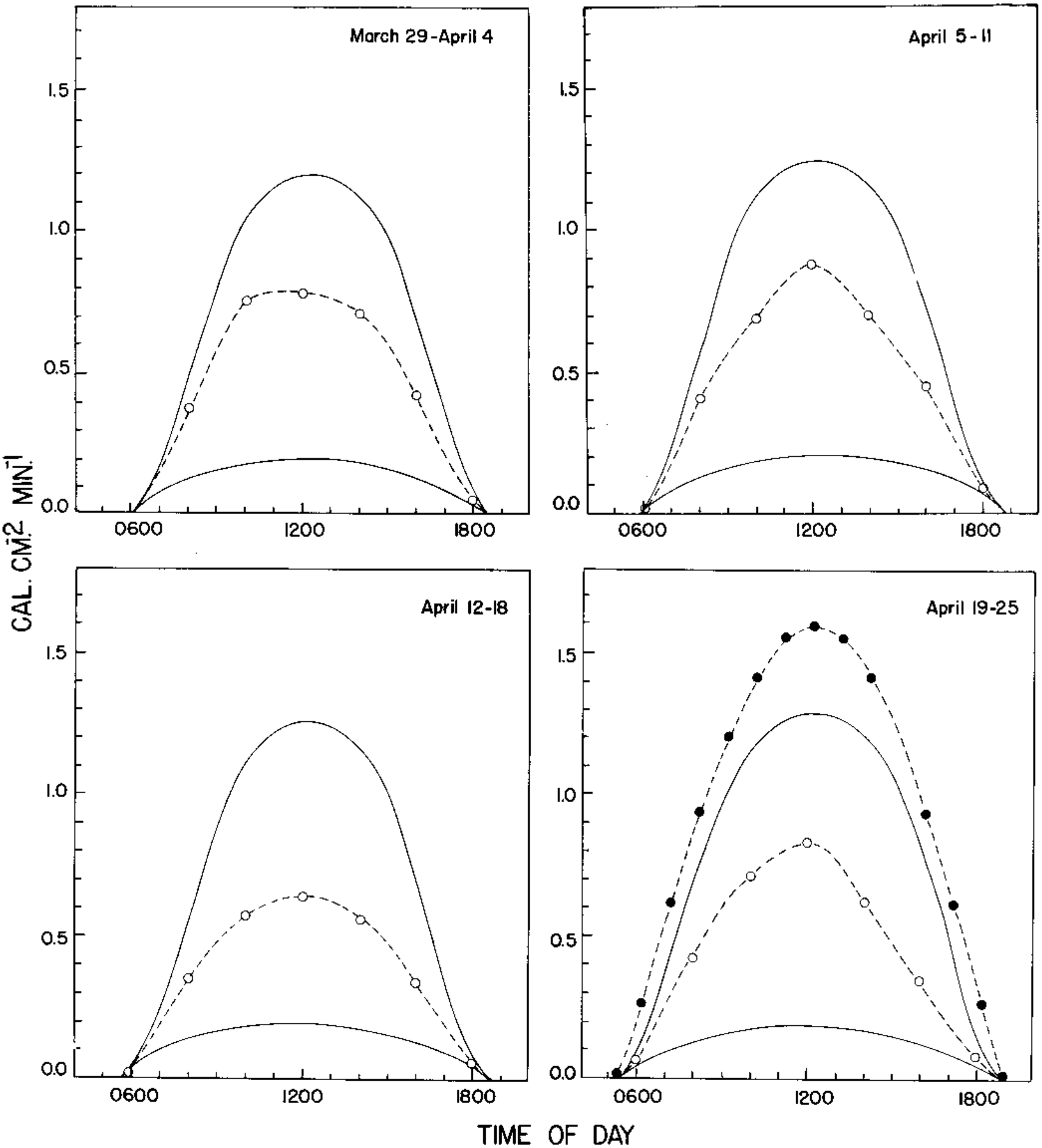



Figure 6. Theoretical and actual radiation at St. Paul during each climatological week. The extraterrestrial radiation received on the 21st day of each month is shown as the uppermost curve in the appropriate week. The other three curves from top to bottom represent, respectively, the average clear-day direct plus diffuse radiation, the average-day direct plus diffuse radiation, and the average clear-day diffuse radiation. Time of day is central standard time.

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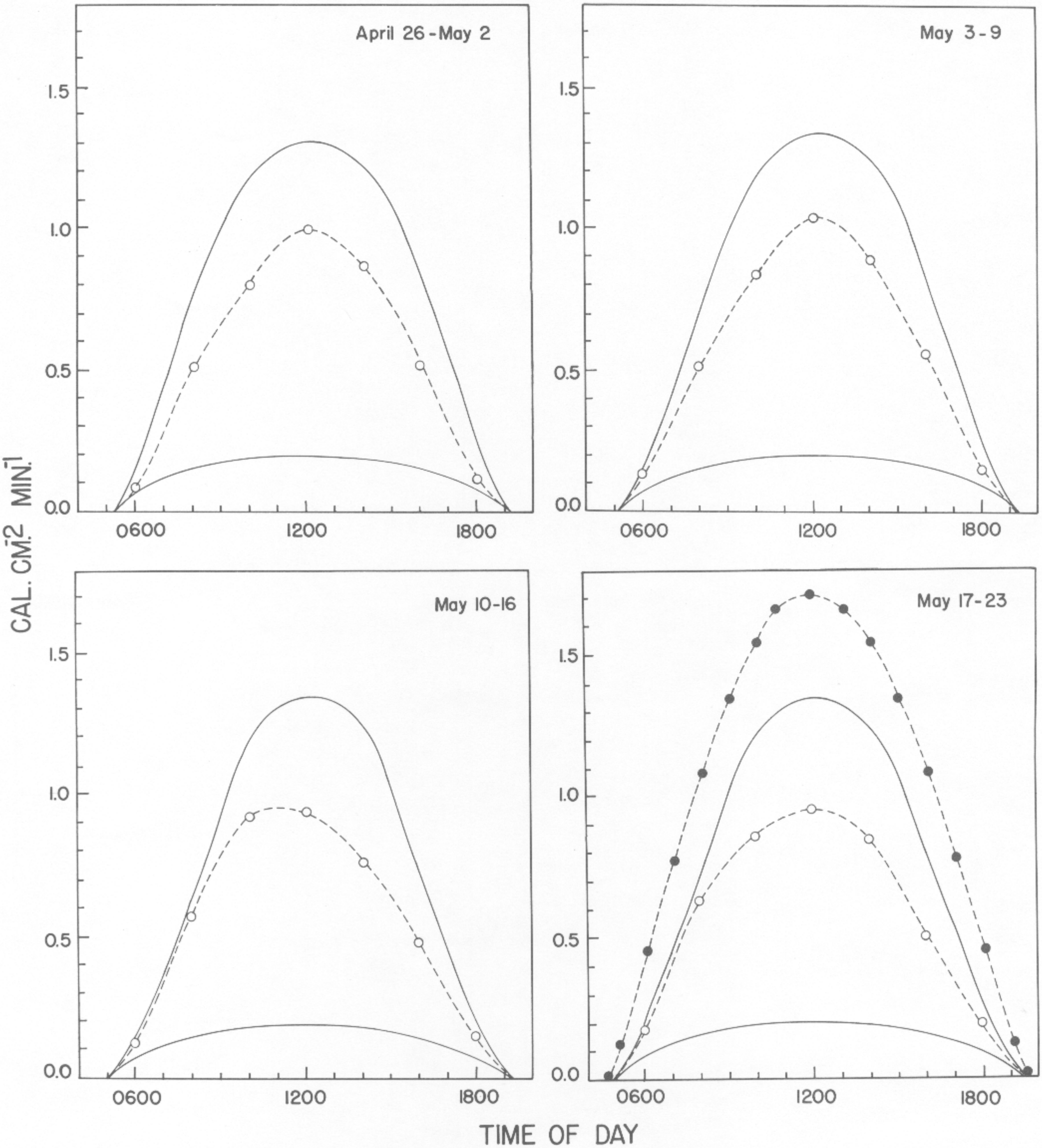


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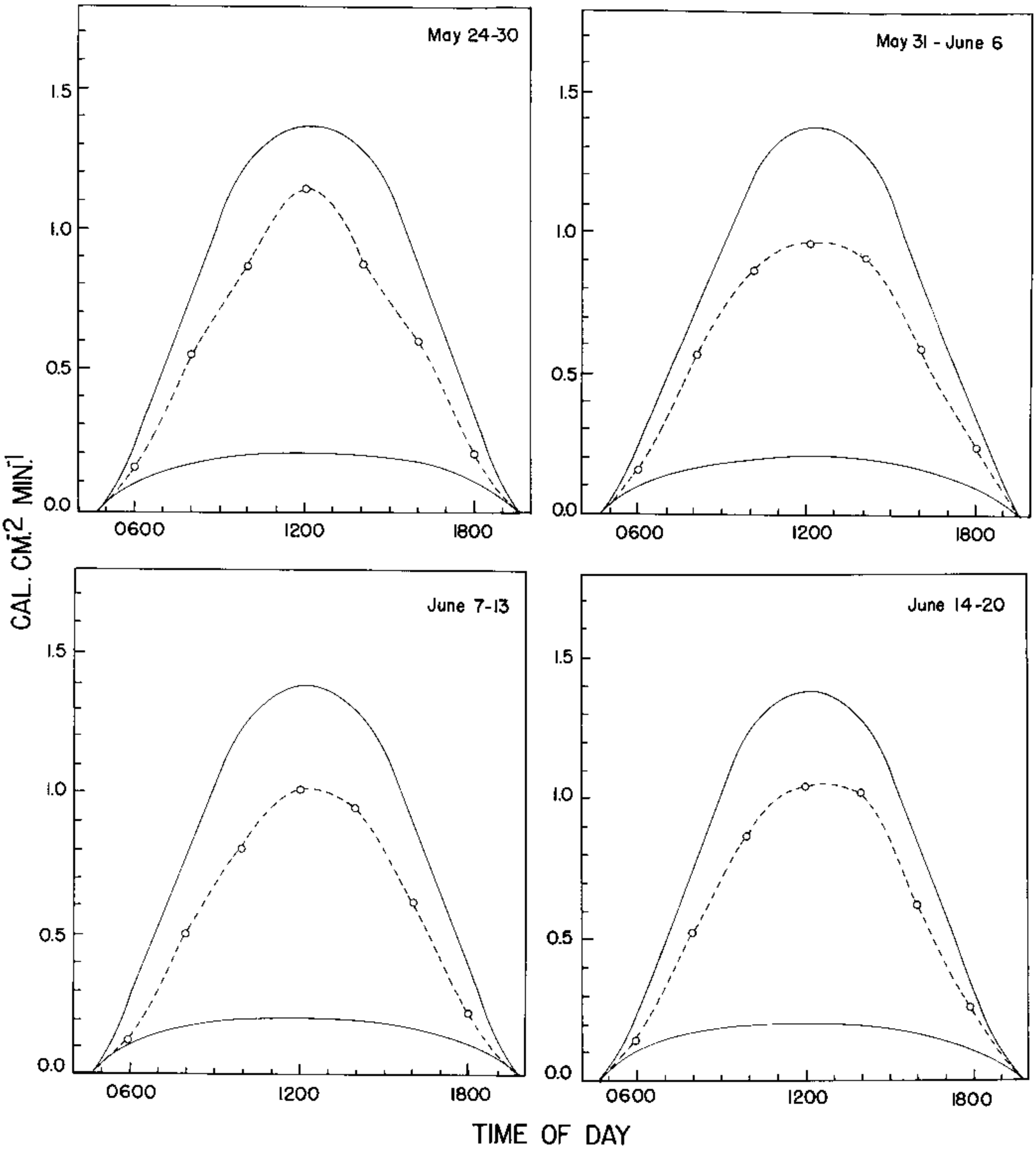


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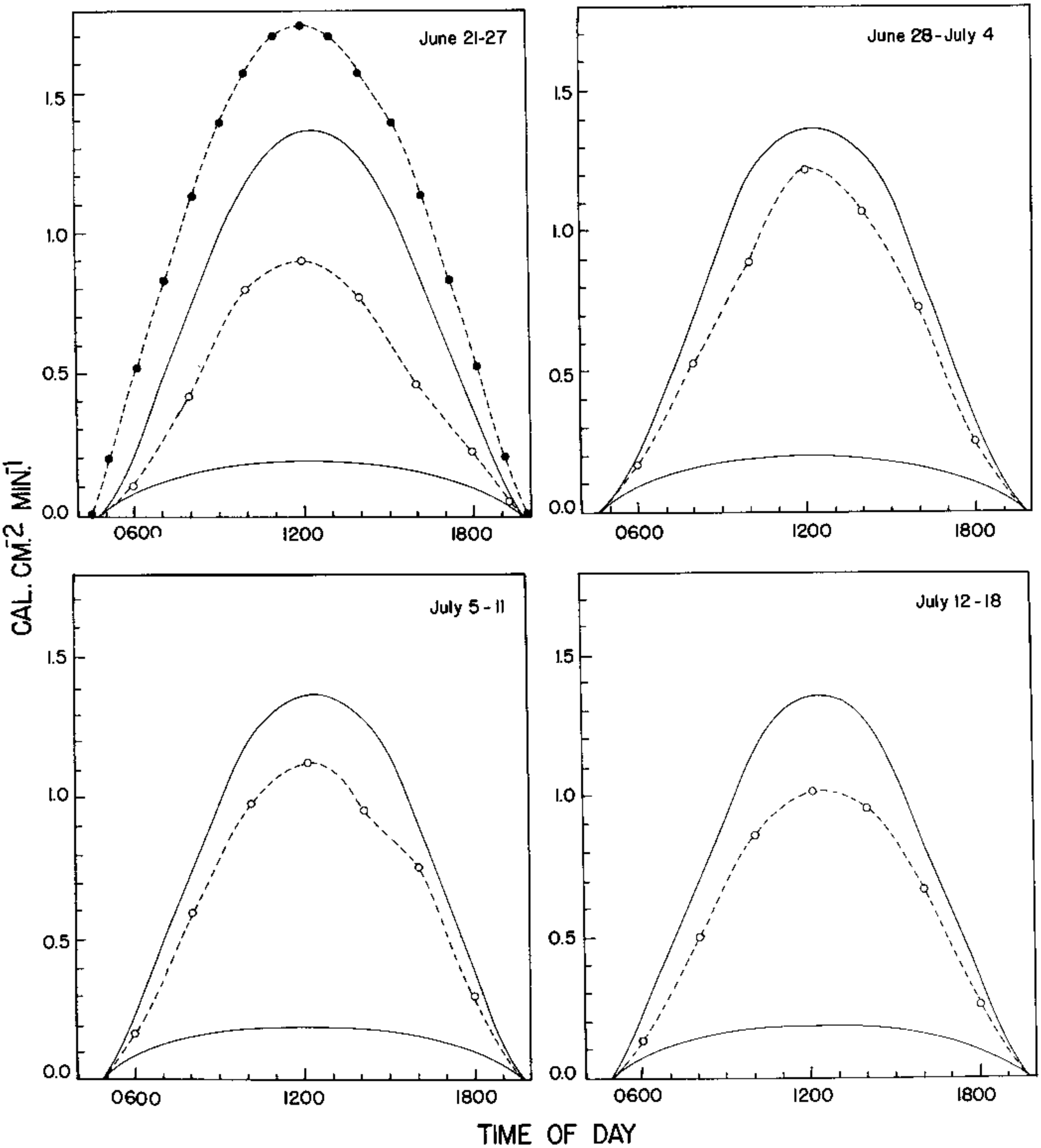


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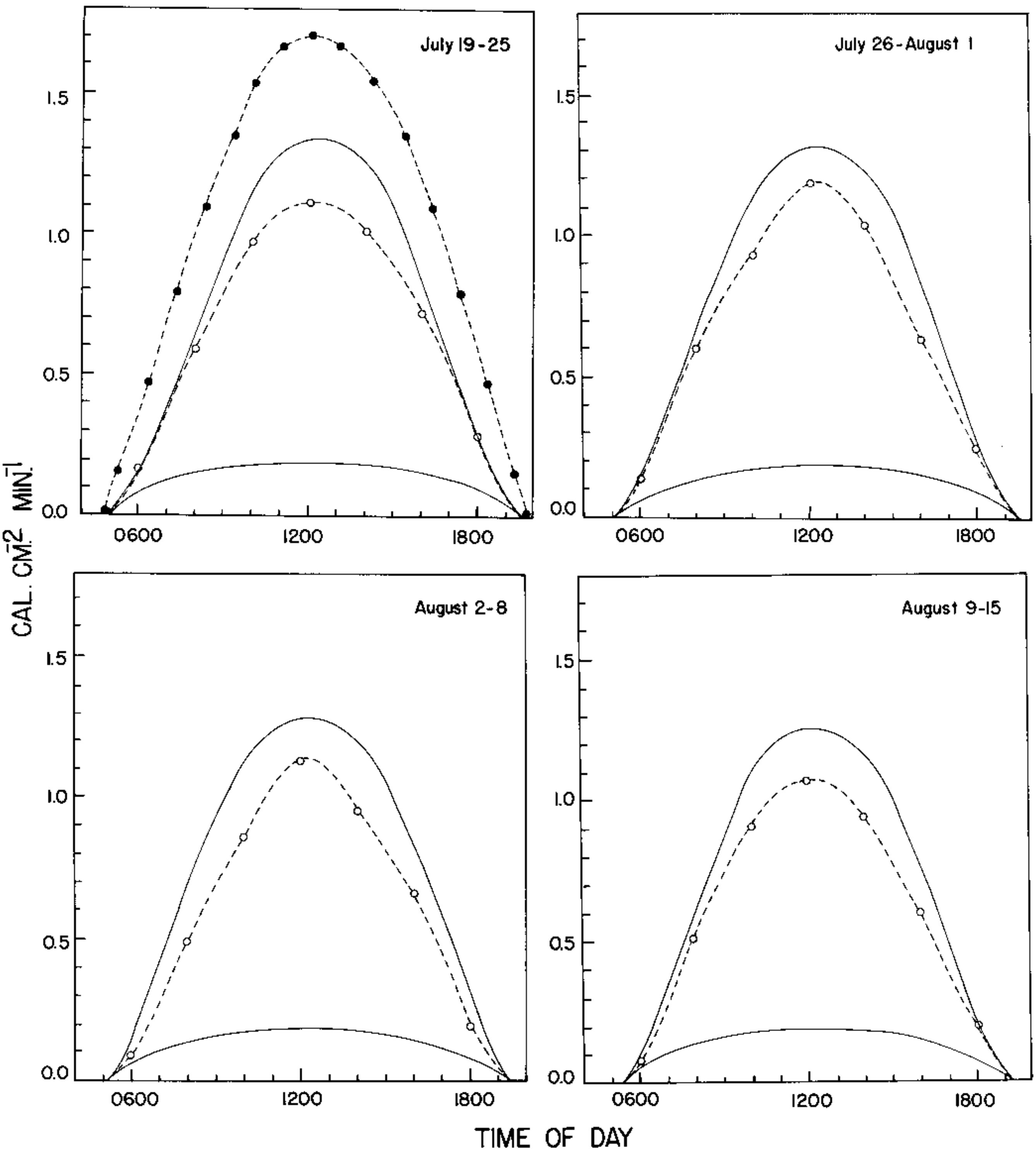


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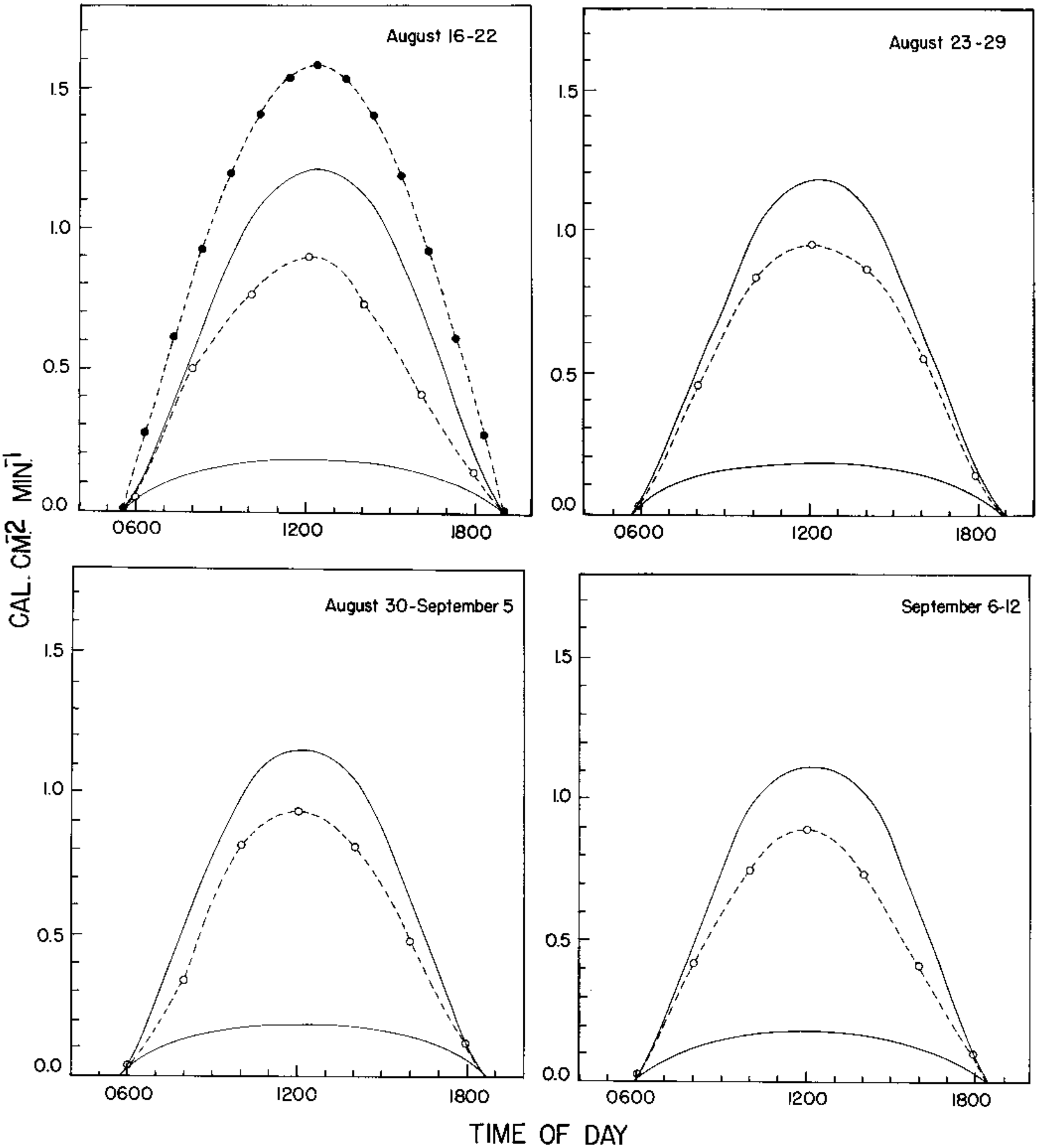


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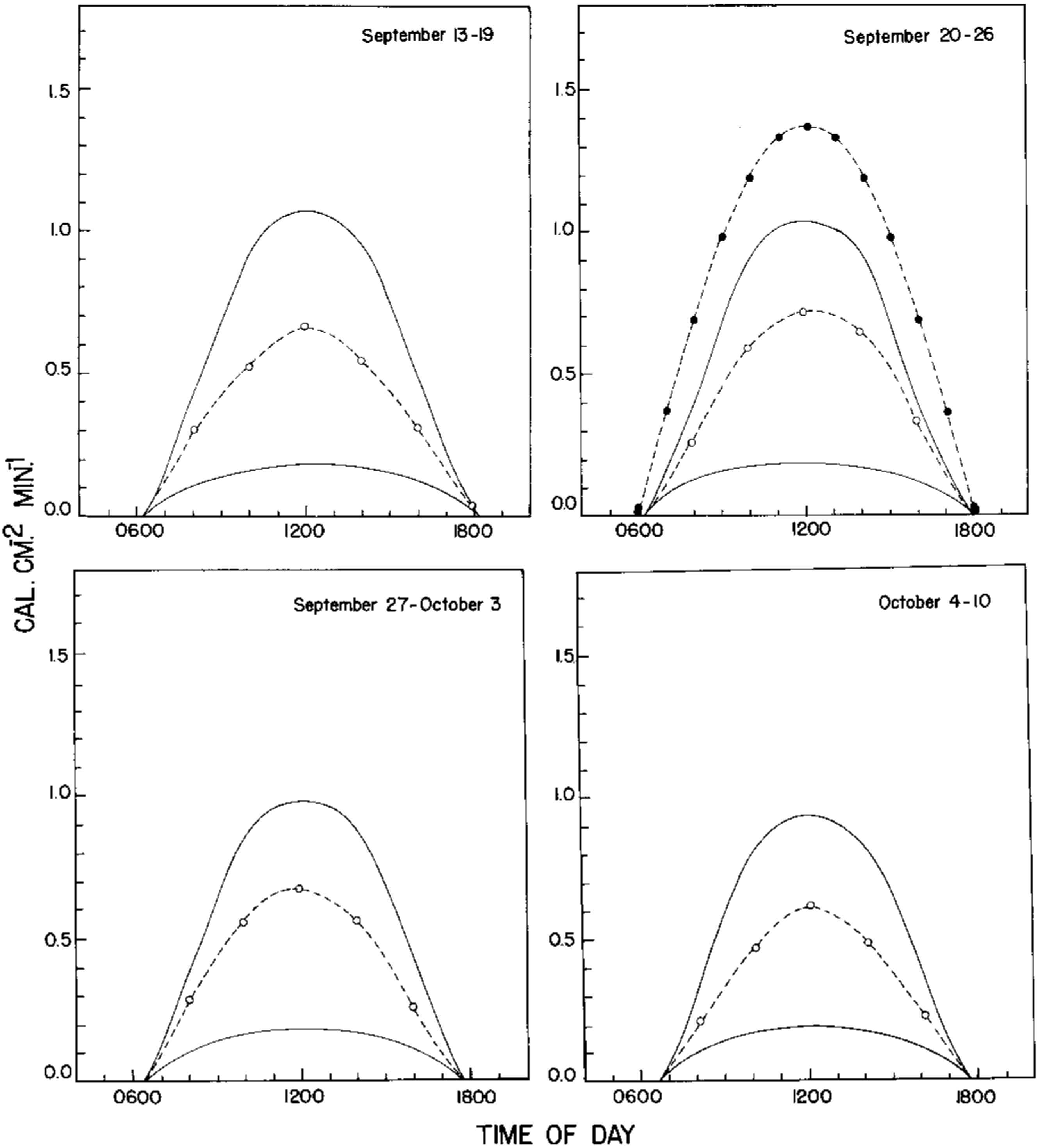


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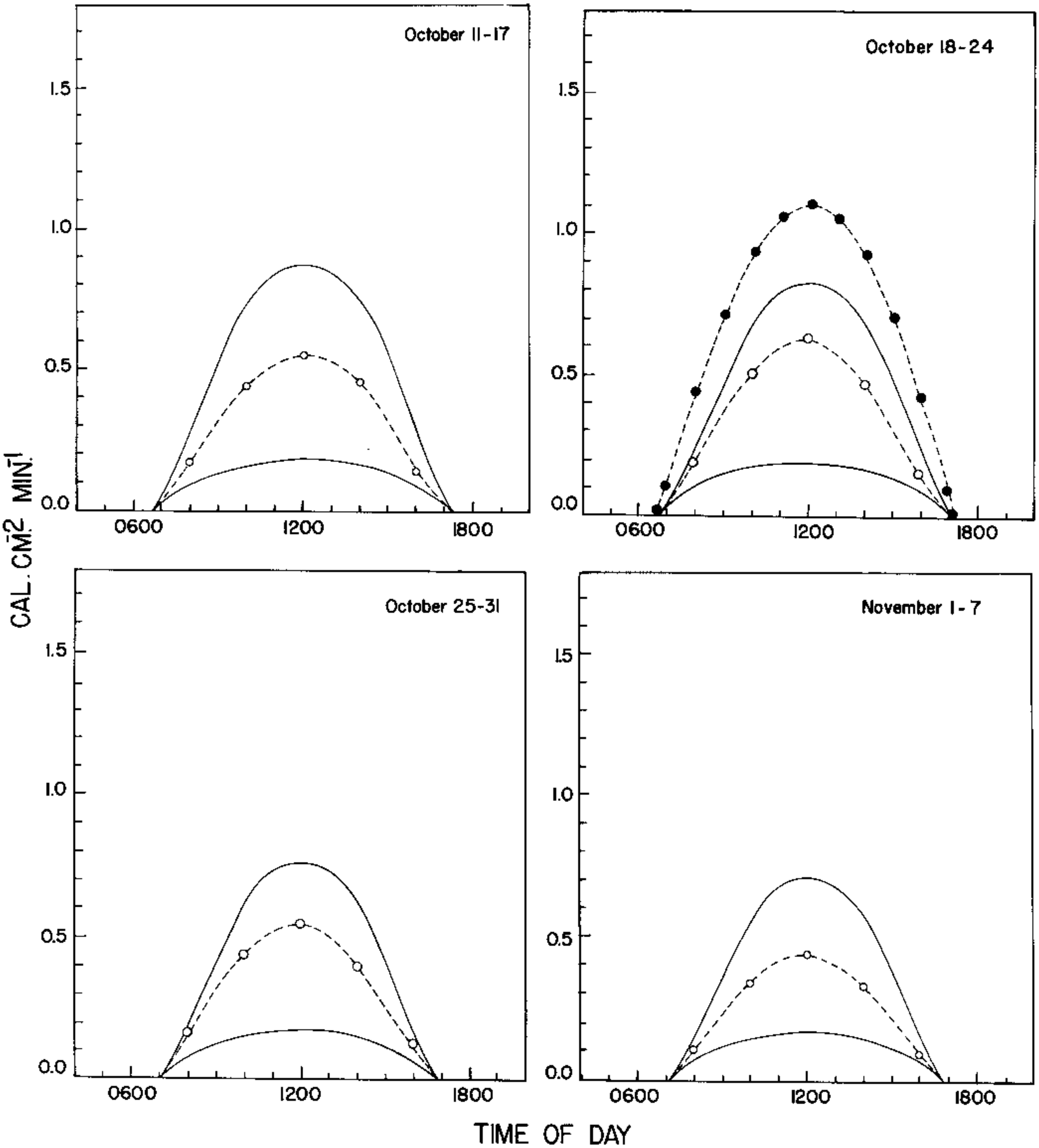


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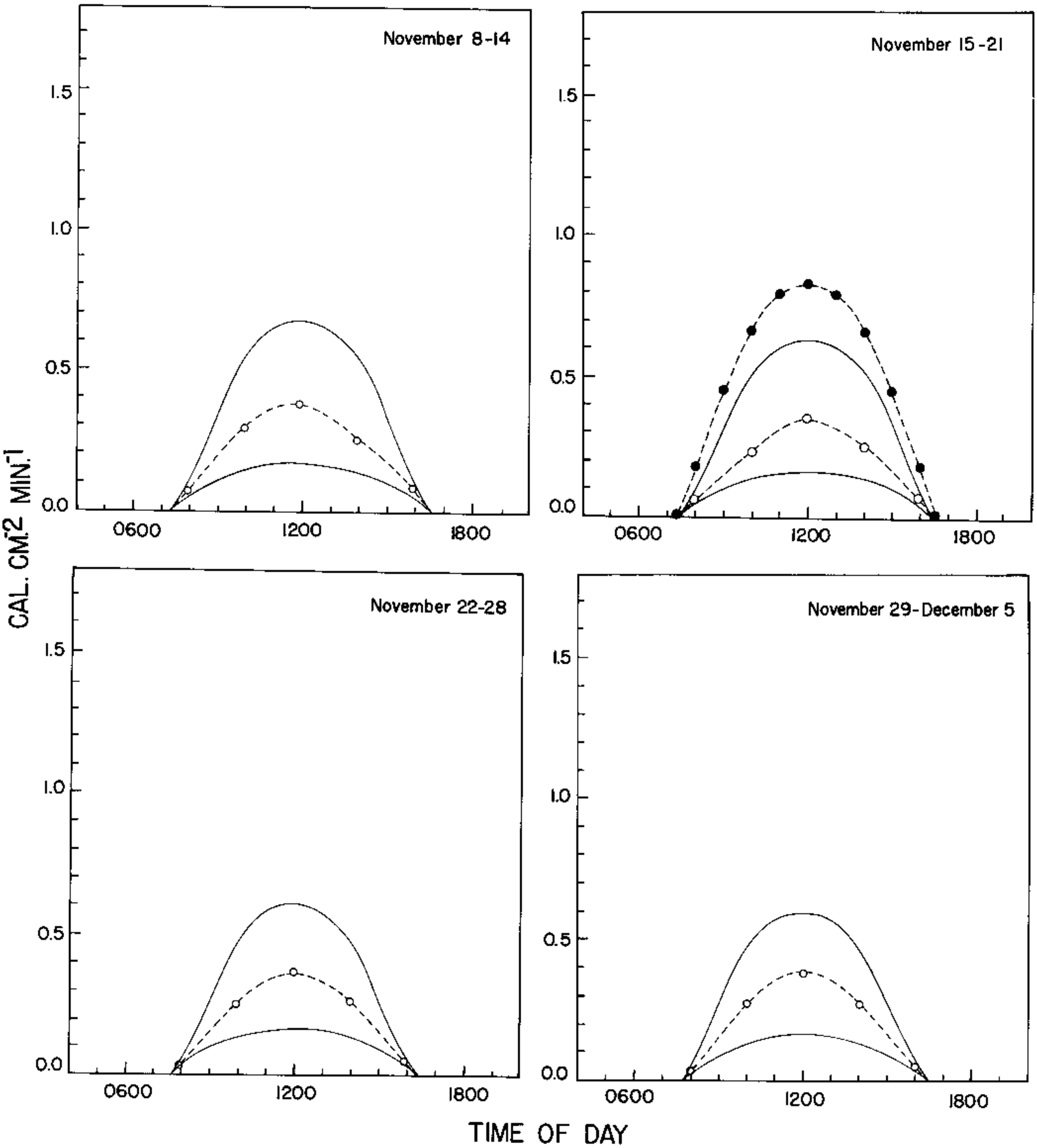


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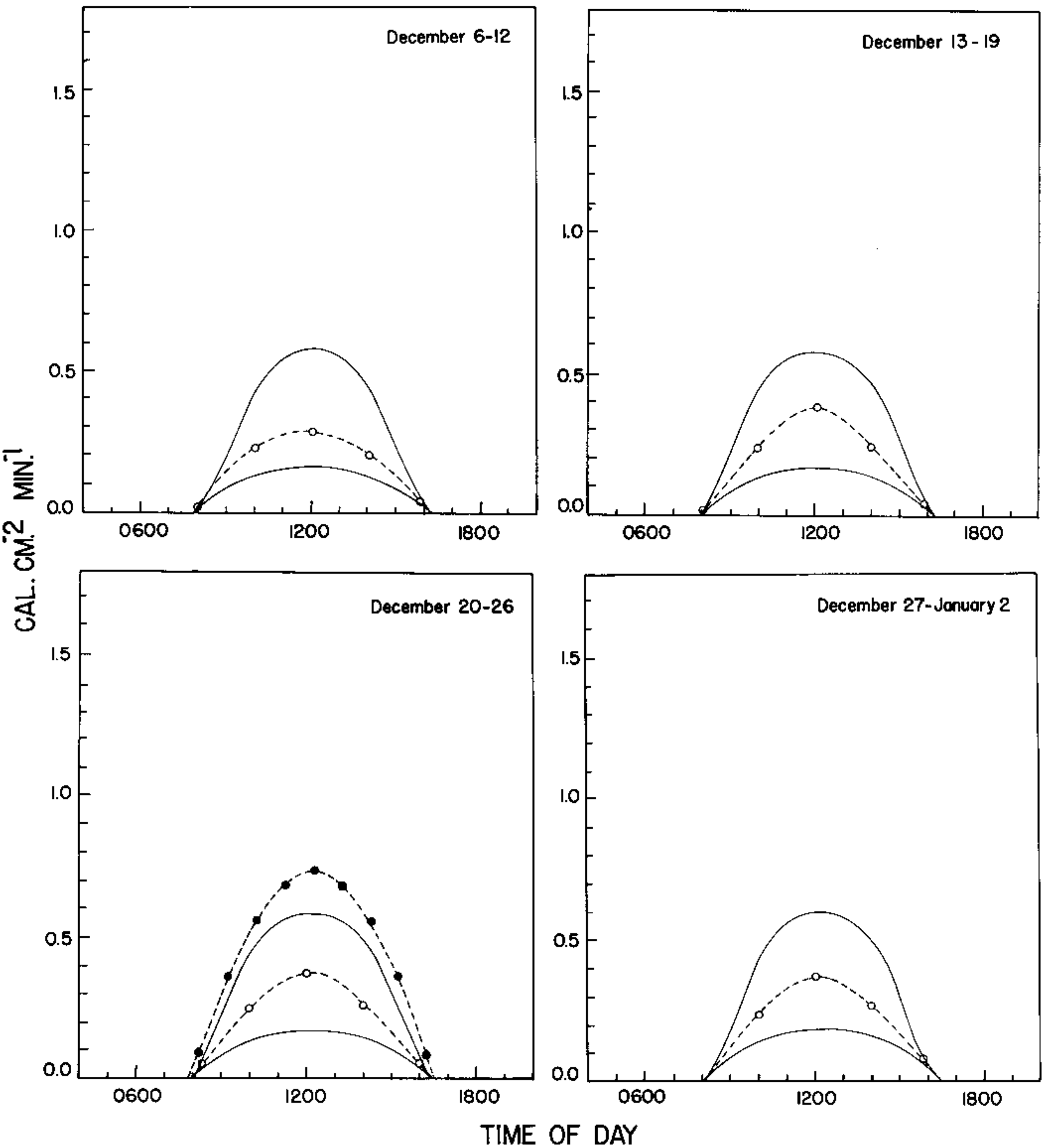


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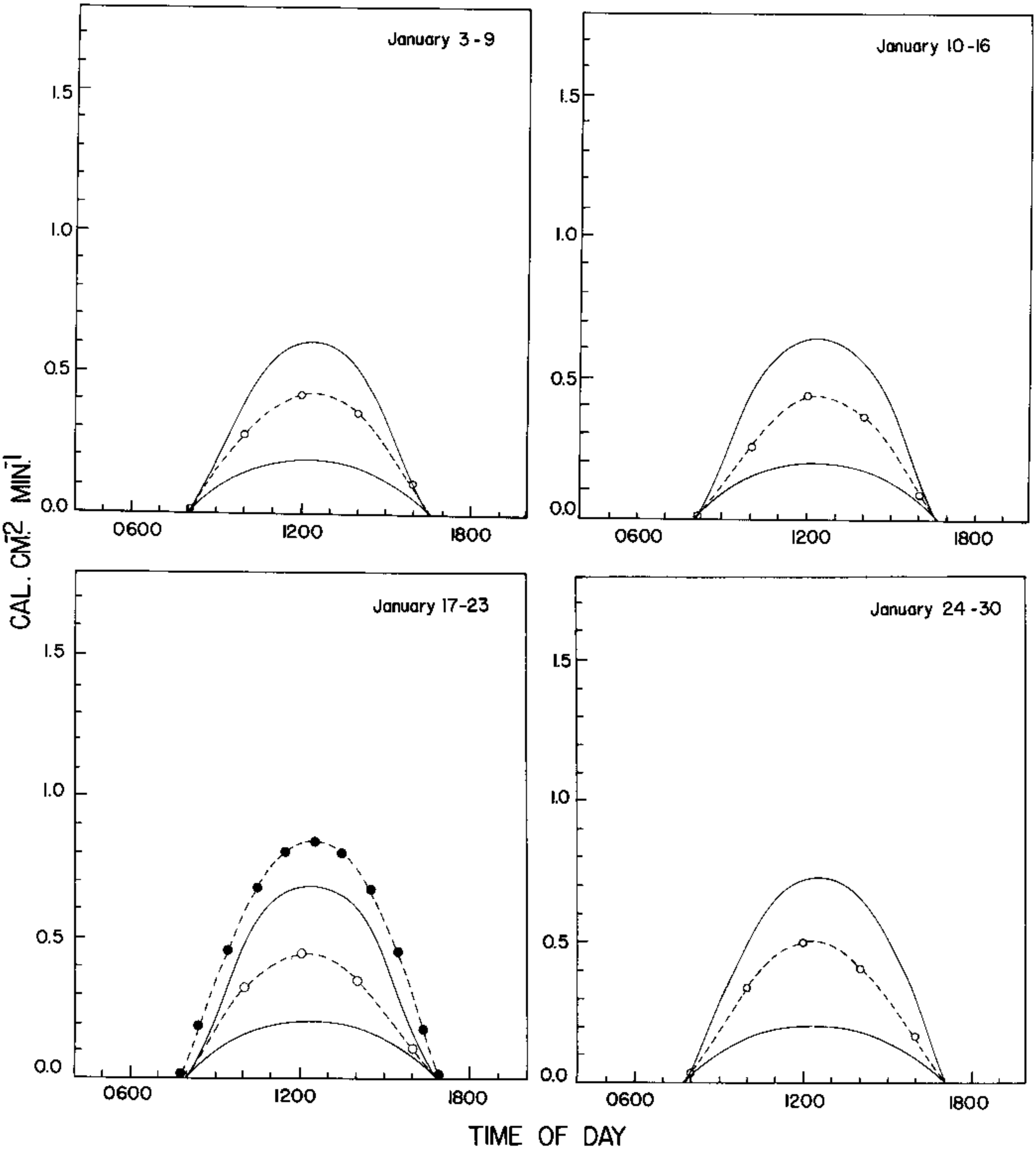


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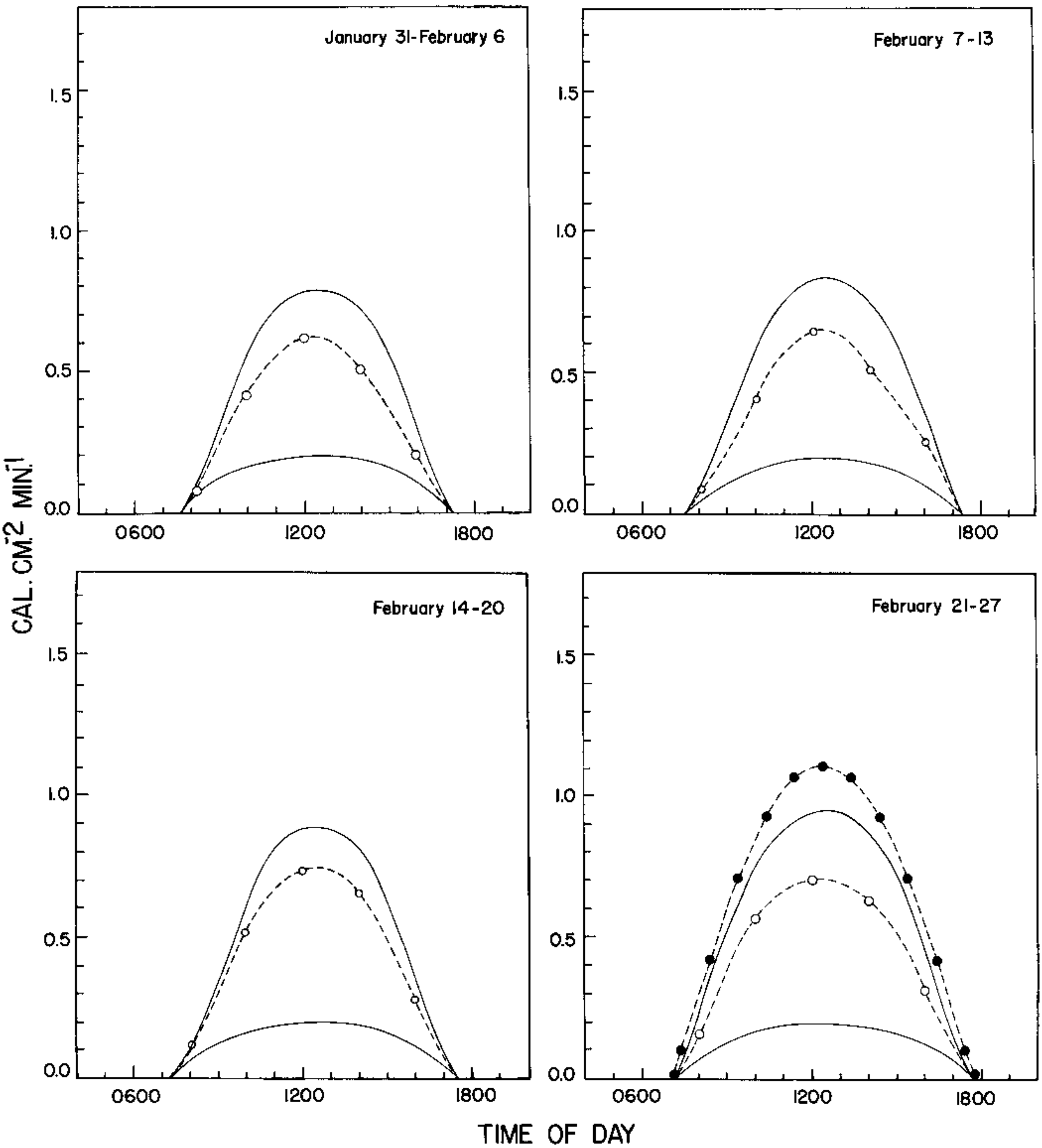


Figure 6. (continued)

Figures 7 and 8 summarize the previous 52 average daily figures. Figure 7 shows the annual march of the radiation received at solar noon. Figure 8 shows the total daily radiation received throughout the year. Only in a few weeks of the year is the average daily noon radiation greater than the direct beam radiation on a clear day, figure 7. On a daily total basis, the average radiation received during the period of record never exceeded the clear-day direct beam total radiation, figure 8.

The clear-day direct plus diffuse radiation values at solar noon (figure 7) will vary probably no more than about 10 percent from the values shown. While the sum of the two components of solar radiation, direct beam and diffuse, may not vary much on a clear day, the individual components do differ greatly as a result of variations in the composition of the atmosphere. For example, during the 5-year period (1965-1969) used in this study, the solar noon diffuse radiation has varied from 30 percent higher to 40 percent lower than the 0.20 cal $cm^{-2}$ $min^{-1}$ shown. The constant value of the solar noon diffuse radiation (D in figure 7) during the year indicates that the drier, clearer atmosphere of winter compensated for the longer atmospheric path of winter radiation.

The ratio of the diffuse (D) to the direct plus diffuse radiation (S + D, figure 7) at noon on a clear day av-

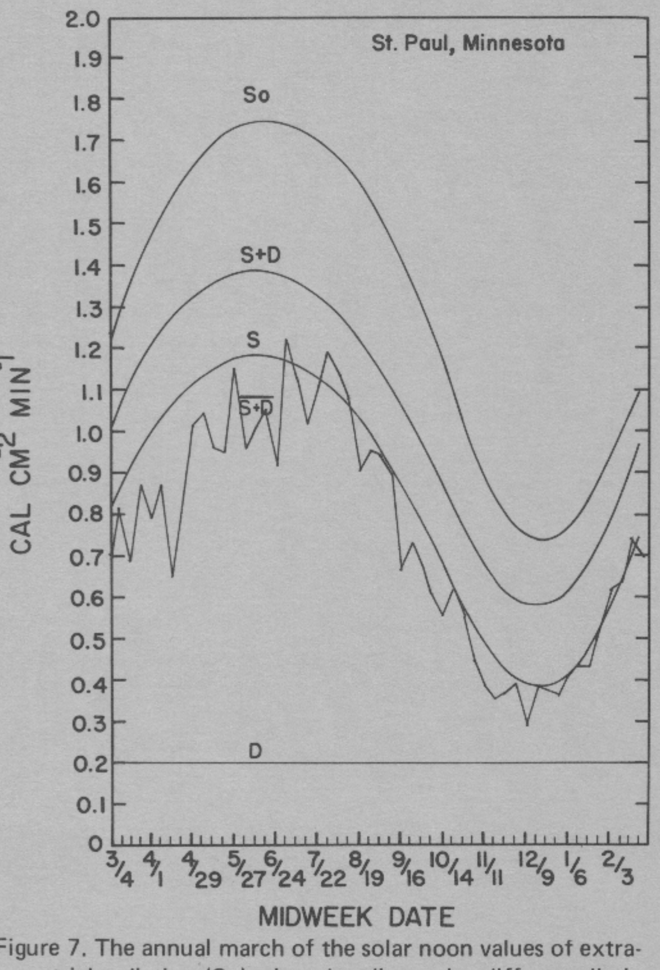


Figure 7. The annual march of the solar noon values of extraterrestrial radiation (So), clear-day direct plus diffuse radiation (S + D), clear-day direct radiation (S), average-day direct plus diffuse radiation ($\overline{S + D}$), and clear-day diffuse radiation (D) at St. Paul.

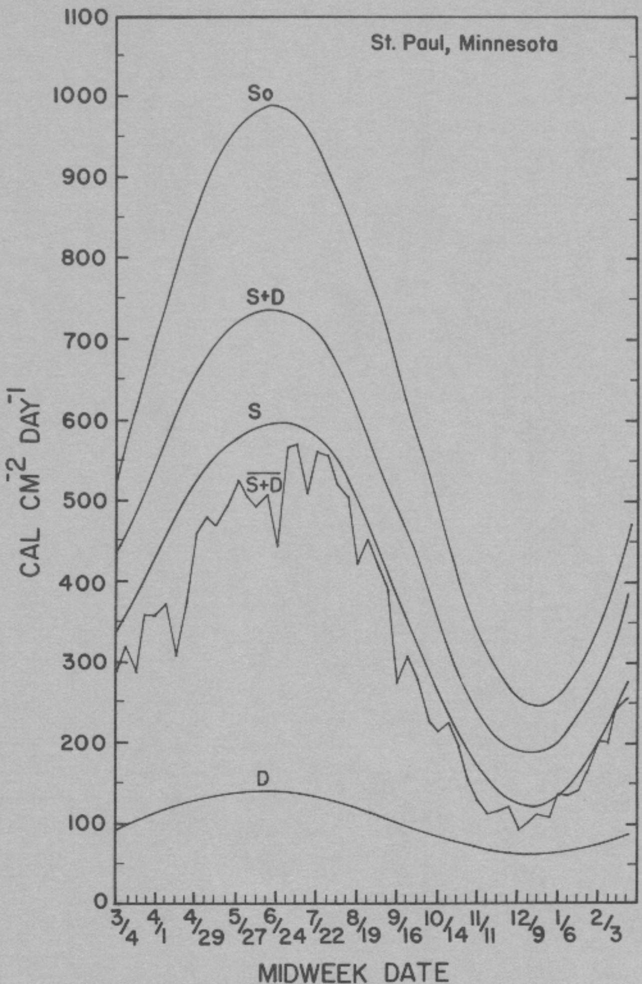


Figure 8. The annual march of the total daily values of extraterrestrial radiation (So), clear-day direct plus diffuse radiation (S + D), clear-day direct radiation (S), average-day direct plus diffuse radiation ($\overline{S + D}$), and clear-day diffuse radiation (D) at St. Paul.

eraged 0.18, 0.14, 0.19, and 0.34 on March 21, June 21, September 23, and December 21, respectively. The ratio of the diffuse to the total daily radiation on clear days for the same four dates averaged 0.21, 0.19, 0.21, and 0.34 (figure 8).

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